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Equivalent circuits in Fourier space for the study of electromagnetic fields

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Abstract. The introduction of properly defined magnetic and electric circuits in Fourier space can give a new powerful technique in studying problems of electromagnetic waves, and a novel view of the behaviour of these waves, in different applications. Using Parseval's identity theorem, one is able to obtain all the possible information for electromagnetic waves by its spatial Fourier transforms. Thus inversion in real space can be avoided completely.

In this paper we present the theory and examples of this new technique in various applications of electromagnetic waves. The coincidence with already existing theoretical formulae or results is evidence that the proposed method is completely equivalent to the classical formulation. But, due to its simplicity this technique could be very useful in studying electromagnetic field problems.

1. Introduction

Spatial Fourier transforms, combined with the derived circuits, have been extensively used to study induction problems in low-frequency applications where the displacement currents are omitted leading to the so-called quasi-stationary formulation (Papageorgiou 1979, Freeman 1968, Freeman and Papageorgiou 1979).

In this paper for higher frequencies, time and space Fourier transforms lead to the idea of introducing two separately excited circuits, one electric and the other magnetic, which are related to the already existing ideas of transverse magnetic and electric fields, respectively. The circuits are not interlinked. The complete solution of the derived circuits can give all the Fourier transformed field components and thus, by inversion, the real space field components can be calculated. Using Parseval's identity formulae, real and reactive powers and radiation patterns can be evaluated in Fourier space. Also, using standard circuit properties, the main electromagnetic field properties can be derived. Resonance in these circuits, for example, is related to surface waves in dielectric layers. Itoh and Menzel (1981) proposed similar ideas for the study of microstrip radiation problems.

In § 2, the Fourier transform method and the basic properties of the derived electric and magnetic circuits are studied. In § 3 the basic electric and magnetic excitations in Fourier space are also given. In § 4 Poynting vector ideas combined with properties of Parseval's identity are used to express power in Fourier space. The radiation pattern function for the far field can be also expressed in Fourier space. Simple examples for the verification of the method are given. Finally, in § 5 the surface modes propagating inside a dielectric slab are shown to be related to conditions of resonance in the corresponding equivalent circuits.

2. Maxwell equations, Fourier transforms and equivalent circuits

Let us consider a homogeneous dielectric layer with electromagnetic parameters ε, μ and σ. The structure is shown in figure 1; the z axis is normal to the layer boundary planes. The electric and magnetic fields E, H, D, B inside the layer are generally functions of the coordinates (x, y, z) and time (t).

In a case of a monochromatic wave every field component $A(x, y, z, t)$ is given by

$$A(x, y, z, t) = \mathrm{Re}[A(x, y, z)\,\mathrm{e}^{\mathrm{i}\omega t}]. \tag{1}$$

Hence its spatial Fourier transform along the x and y axes will be

$$\mathring{A}(\alpha, \beta, z) = \iint_{-\infty}^{\infty} A(x, y, z)\,\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{e}^{-\mathrm{i}\beta y}\,\mathrm{d}x\,\mathrm{d}y \tag{2}$$

and inversely

$$A(x, y, z) = \iint_{-\infty}^{\infty} \mathring{A}(\alpha, \beta, z)\,\mathrm{e}^{\mathrm{i}\alpha x + \mathrm{i}\beta y}\left(\frac{\mathrm{d}\alpha}{2\pi}\right)\left(\frac{\mathrm{d}\beta}{2\pi}\right). \tag{3}$$

Fourier transforming the Maxwell equations, the following system of six equations is obtained. For the sake of simplicity, we will use the symbol $\mathring{A}$ for the Fourier transform of the field component A.

$$\begin{aligned}
&\frac{\mathrm{i}\beta}{\mu}\mathring{B}_z - \frac{\partial \mathring{H}_y}{\partial z} = (\sigma + \mathrm{i}\omega\varepsilon)\mathring{E}_x && \frac{\partial \mathring{H}_x}{\partial z} - \frac{\mathrm{i}\alpha}{\mu}\mathring{B}_z = (\sigma + \omega\varepsilon)\mathring{E}_y \\
&\mathrm{i}\alpha\mathring{H}_y - \mathrm{i}\beta\mathring{H}_x = \left(\frac{\sigma}{\varepsilon} + \mathrm{i}\omega\right)\mathring{D}_z && \frac{\mathrm{i}\beta}{\varepsilon}\mathring{D}_z - \frac{\partial \mathring{E}_y}{\partial z} = -\mathrm{i}\omega\mu\mathring{H}_x \\
&\frac{\partial \mathring{E}_x}{\partial z} - \frac{\mathrm{i}\alpha}{\varepsilon}\mathring{D}_z = -\mathrm{i}\omega\mu\mathring{H}_y && \mathrm{i}\alpha\mathring{E}_y - \mathrm{i}\beta\mathring{E}_x = -\mathrm{i}\omega\mathring{B}_z.
\end{aligned} \tag{4}$$

From Maxwell's equations, on a plane boundary between successive planar layers the following components are continuous:
(a) the tangential electric and magnetic field components E_x, E_y, H_x, H_y and
(b) the normal components of the vectors $\partial \boldsymbol{B}/\partial t$ and $\boldsymbol{j} + \partial \boldsymbol{D}/\partial t$.
Thus, in Fourier space the following components will also be continuous: $\mathring{E}_x$, $\mathring{E}_y$, $\mathring{H}_x$, $\mathring{H}_y$ and $\mathrm{i}\omega\mathring{B}_z$, $(\sigma/\varepsilon + \mathrm{i}\omega)\mathring{D}_z$ respectively.

Of course the boundary must be free, i.e. electric or magnetic currents or charges must not exist on it.

We define the following functions in Fourier space:

Magnetic voltage = $V_{\mathrm{M}} = \alpha\mathring{H}_x + \beta\mathring{H}_y$ Magnetic current = $I_{\mathrm{M}} = \omega\mathring{B}_z = \alpha\mathring{H}_y - \beta\mathring{H}_x$

Electric voltage = $V_{\mathrm{E}} = \alpha E_x + \beta E_y$

Electric current = $I_{\mathrm{E}} = (\omega - \mathrm{i}\sigma/\varepsilon)D_z = \beta\mathring{E}_x - \alpha\mathring{E}_y$. (5)

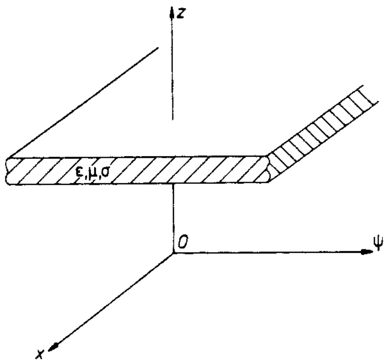

Figure 1. A planar layer with a stationary frame of reference ($Oxyz$).

These are the fundamental magnitudes for the construction of the equivalent circuits. These are functions of (α, β, z) and their units are A, V and V, A, respectively. By inversion the field components can be expressed in terms of V_M, I_M, V_E, I_E.

The previous system (4) can now be written as follows:

$$\partial V_M/\partial z = -(\gamma^2/\mathrm{i}\omega\mu)I_M \qquad \partial I_M/\partial z = -\mathrm{i}\omega\mu V_M \tag{6}$$

and

$$\partial V_E/\partial z = -[\gamma^2/(\sigma+\mathrm{i}\omega\varepsilon)]I_E \qquad \partial I_E/\partial z = -(\sigma+\mathrm{i}\omega\varepsilon)V_E \tag{7}$$

where

$$\gamma^2 = \alpha^2 + \beta^2 - \varepsilon\mu\omega^2 + \mathrm{i}\mu\sigma\omega. \tag{8}$$

The expressions (6) and (7) are the differential equations of a transmission line which is magnetic or electric, respectively. Both lines have the same propagation factor γ (with $\mathrm{Re}(\gamma) \geqslant 0$), but different characteristic impedances, $z_M = \gamma/\mathrm{i}\omega\mu$, and $z_E = \gamma'/(\sigma+\mathrm{i}\omega\varepsilon)$. These lines are not interlinked and can be treated separately.

An equivalent circuit of such a line is given in figure 2, where

$$z_S = z\tanh(\gamma d/2) \qquad z_P = z/\sinh(\gamma d) \tag{9}$$

and z, γ, d are respectively the characteristic impedance, the propagation factor and the thickness of the layer.

By definition, the voltages and currents at the interfaces are continuous, so the problem of successive planar layers leads to a connection of their equivalent magnetic and electric circuits, in cascade.

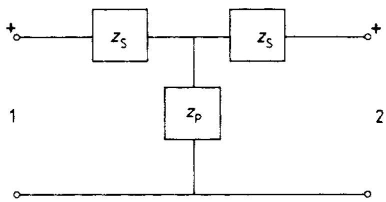

Figure 2. The equivalent T-circuit of an electric line of length d.

3. Boundary conditions and excitation sources

Let us now examine the case of a boundary z = constant where a surface current sheet exists on it with density $\boldsymbol{j}$. If $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ are the magnetic fields just above and below the boundary and $\boldsymbol{n}$ is the normal unit vector, the following jump condition is true at the boundary:

$$\boldsymbol{n} \times (\boldsymbol{H}_2 - \boldsymbol{H}_1) = \boldsymbol{j}. \tag{10}$$

Thus, Fourier transforming and rearranging terms, the following algebraic relation holds:

$$(\alpha \mathring{H}_x + \beta \mathring{H}_y)_2 - (\alpha \mathring{H}_x + \beta \mathring{H}_y)_1 = \alpha \mathring{j}_y - \beta \mathring{j}_x \tag{11}$$

where $\mathring{H}_x$, $\mathring{H}_y$, $\mathring{j}_x$, $\mathring{j}_y$ are functions of α and β. Hence the function $V_{\mathrm{M}} = \alpha \mathring{j}_y - \beta \mathring{j}_x$ represents a magnetic voltage source connected between the two magnetic lines separated by the previous boundary.

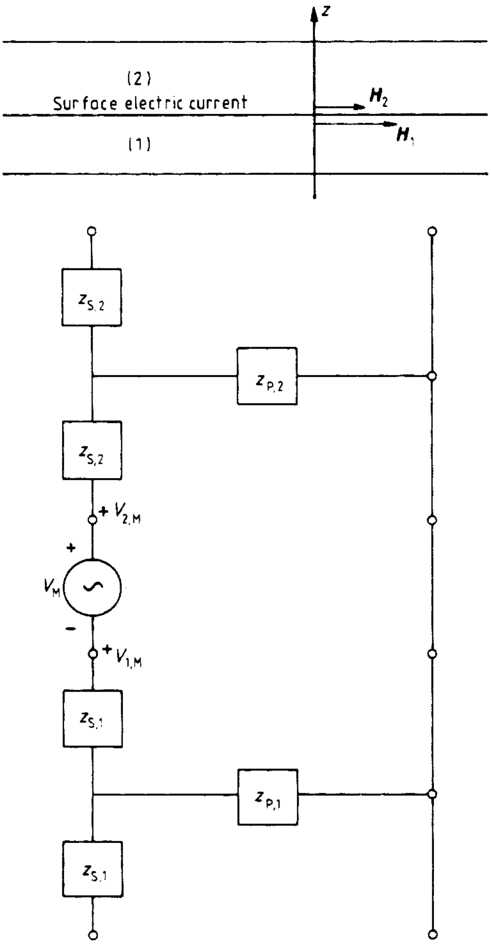

Figure 3. Two successive layers with a current sheet on their boundary surface, and their equivalent magnetic circuit in Fourier space.

Let us now consider a boundary plane charge sheet with surface charge density $\rho_S(x, y)$. If $D_{z,1}$ and $D_{z,2}$ are the electric displacement normal components just above and below the boundary, the following jump condition is true at the boundary:

$$D_{z,1} - D_{z,2} = \rho_S(x, y). \tag{12}$$

Fourier transforming and using the definition for the electric current (for $\sigma = 0$), the following equation holds:

$$I_{E,1} - I_{E,2} = \omega \mathring{\rho}_S. \tag{13}$$

Hence the function $I_E = \omega \mathring{\rho}_S$ represents an electric current source connected in parallel between the two electric lines separated by the previous boundary. In general, a current sheet with current density components $J_x(x, y)$, $J_y(x, y)$ has also a charge density $\rho_S(x, y)$.

The continuity equation after Fourier transforming becomes

$$\alpha \mathring{J}_x + \beta \mathring{J}_y = -\omega \mathring{\rho}_S \tag{14}$$

where $\mathring{J}_x$, $\mathring{J}_y$, $\mathring{\rho}_S$ are functions of α and β. Hence the function $I_E = -(\alpha \mathring{J}_x + \beta \mathring{J}_y)$ represents, as stated previously, an electric current source.

Let us now consider the case of a Hertz infinitesimal dipole of length $\Delta l(\Delta x, \Delta y)$ and current I, tangentially oriented on a boundary plane at the point (x, y). It can be easily shown that it is equivalent to a surface current sheet, so that this tangential dipole can be simulated by a voltage magnetic source together with a current electric source.

A thin filamentary planar current conductor, on a plane boundary, can be considered as the distribution of Hertz dipoles, hence is equivalent to a voltage magnetic source

$$V_M = \int_l I(x, y) \exp(\mathrm{i}\alpha x + \mathrm{i}\beta y)(\alpha \, \mathrm{d}y - \beta \, \mathrm{d}x) \tag{15}$$

and to a current electric source

$$I_E = -\int_l I(x, y) \exp(\mathrm{i}\alpha x + \mathrm{i}\beta y)(\alpha \, \mathrm{d}x + \beta \, \mathrm{d}y). \tag{16}$$

The integrals are calculated along the length l of the conductor.

As expected, the second integral for a closed thin filamentary loop with constant current is zero. Using the duality principle we can prove that a surface magnetic current sheet with components in Fourier space $J_{M,x}$ and $J_{M,y}$ will be represented by a voltage electric source

$$V_E = \alpha \mathring{J}_{M,y} - \beta \mathring{J}_{M,x} \tag{17}$$

and a current magnetic source

$$I_M = -(\alpha \mathring{J}_{M,x} + \beta \mathring{J}_{M,y}). \tag{18}$$

For the former case of a magnetic current sheet there is an important application, the calculation of the equivalent excitation of a normal Hertz electric dipole at a plane boundary. As is already known this normal infinitesimal dipole of length Δl and current I is electromagnetically equivalent to a small magnetic current loop with surface Δs and a constant magnetic current

$$I_{\mathrm{M}} = I\Delta l/\varepsilon\omega\Delta s. \tag{19}$$

Combining the previous ideas, we conclude that an electric normal dipole will be equivalent to an electric voltage source given by

$$V_{\mathrm{E}} = (I\Delta l/\varepsilon\omega\Delta s)\exp(\mathrm{i}\alpha x + \mathrm{i}\beta y)(\alpha\,\mathrm{d}y - \beta\,\mathrm{d}x) \tag{20}$$

where the integration takes place on an infinitesimal loop with area Δs. In order to proceed further we can consider a small square loop with side Δx. Hence, integrating, the following result is derived:

$$V_{\mathrm{E}} = \mathrm{i}\frac{I\Delta l}{\varepsilon\omega}\,\frac{\sin(\alpha\Delta x/2)\sin(\beta\Delta x/2)}{(\alpha\Delta x/2)(\beta\Delta x/2)}(\alpha^2+\beta^2). \tag{21}$$

Hence for all wavenumbers where $\alpha\Delta x/2$ and $\beta\Delta x/2$ are very small compared with unity

$$V_{\mathrm{E}} = \mathrm{i}(I\Delta l/\varepsilon\omega)(\alpha^2+\beta^2). \tag{22}$$

4. Calculation of the radiation pattern

As a first application of the proposed technique let us give formulae for the calculation of the radiation pattern in Fourier space. Rhodes (1966) has tackled the problem in a similar way for planar radiating structures.

The first step is the evaluation of the power crossing a plane normal to the z axis, in terms of Fourier space components. This power can be calculated in terms of a Poynting vector, as

$$S = \iint_{-\infty}^{\infty} (\boldsymbol{E}\times\boldsymbol{H})_z\,\mathrm{d}x\,\mathrm{d}y. \tag{23}$$

For a monochromatic excitation, the total complex power $\mathring{S}$ is given by

$$\mathring{S} = P + \mathrm{i}Q = \tfrac{1}{2}\iint_{-\infty}^{\infty} (\boldsymbol{E}\times\boldsymbol{H}^*)_z\,\mathrm{d}x\,\mathrm{d}y \tag{24}$$

where P = real power, Q = reactive power and the symbol * means conjugate of a complex number. Using Parseval's identity theorem, we can evaluate the same power $\mathring{S}$ in Fourier space, as follows:

$$\mathring{S} = \tfrac{1}{2}\iint_{-\infty}^{\infty} (\mathring{E}_x\mathring{H}_y^* - \mathring{E}_y\mathring{H}_x^*)\frac{\mathrm{d}\alpha}{2\pi}\,\frac{\mathrm{d}\beta}{2\pi} \tag{25}$$

where the components $\mathring{E}_x$, $\mathring{E}_y$, $\mathring{H}_x^*$, $\mathring{H}_y^*$ are functions of α and β. If the field components are expressed in terms of V_{M}, V_{E}, I_{M}, I_{E}

$$\mathring{S} = \mathring{S}_{\mathrm{E}} + \mathring{S}_{\mathrm{M}} \tag{26}$$

where

$$\mathring{S}_{\mathrm{E}} = \frac{1}{8\pi^2} \iint\limits_{-\infty}^{\infty} \frac{V_{\mathrm{E}} I_{\mathrm{E}}^{*}}{\alpha^2+\beta^2} \, \mathrm{d}\alpha \, \mathrm{d}\beta \tag{27}$$

and S_{M} is defined analogously.

Let us consider that above (or below) the plane under consideration, there is a homogeneous infinitely thick layer, which is represented by its characteristic impedances in both electric and magnetic equivalent circuits, i.e.

$$z_{\mathrm{E}} = \gamma/(\sigma + \mathrm{i}\varepsilon\omega), \qquad z_{\mathrm{M}} = \gamma/\mathrm{i}\omega\mu. \tag{28}$$

If I_{E} and I_{M} are the electric and magnetic currents respectively, the power which crosses the plane can be written as

$$\mathring{S} = \frac{1}{8\pi^2} \iint\limits_{-\infty}^{\infty} \left(\frac{I_{\mathrm{E}}^2}{\alpha^2+\beta^2} z_{\mathrm{E}} + \frac{I_{\mathrm{M}}^2}{\alpha^2+\beta^2} z_{\mathrm{M}} \right) \mathrm{d}\alpha \, \mathrm{d}\beta. \tag{29}$$

We consider first the case of an infinitely thick layer without ohmic losses ($\sigma = 0$, everywhere); in that case

$$\gamma = (\lambda^2 - K^2)^{1/2} \qquad K^2 = \omega^2 \mu\varepsilon \qquad \lambda^2 = \alpha^2 + \beta^2 \qquad K, \lambda \geqslant 0. \tag{30}$$

If now $\lambda \geqslant K$, γ is real, z_{E} and z_{M} are pure imaginary and $\mathring{S}$ is pure imaginary. This means that in this case, there is no radiated power in the space. On the other hand, if $\lambda < K$, then $\gamma = \mathrm{i}(K^2 - \lambda^2)^{1/2} = \mathrm{i}C$ (pure imaginary), z_{E}, z_{M} are reals and $\mathring{S}$ is real. In this case, all the power is radiated in the space.

Considering now a lossy medium ($\sigma > 0$); z_{E} and z_{M} are for every value of λ complex numbers and thus P, Ω exist for all values of λ. But, again for the usual case when σ is relatively small (i.e. $\sigma \ll \varepsilon\omega$), the main part of $\mathring{S}$ is real for $\lambda < K$, while the main part of $\mathring{S}$ is imaginary for $\lambda \geqslant K$.

The power formula can give the radiated and non-radiated powers respectively, in the whole space due to a general excitation. With a simple transformation, one is able to obtain the radiation pattern in the far field. A harmonic plane wave with wavenumber K, and wavenumbers along the x and y axes α and β respectively, will have a propagation vector $\boldsymbol{K}$ which will be directed along the φ, θ directions in spherical coordinates, shown in figure 4. The following formulae are true:

$$\alpha = K \sin\theta \cos\varphi \qquad \beta = K \sin\theta \sin\varphi \qquad c = K \cos\theta \qquad \text{and } \lambda = K \sin\theta \leqslant K. \tag{31}$$

The real power P which is calculated by the expression (29) can now be written as

$$P = \frac{K}{8\pi^2} \int_0^{\pi/2} \int_0^{2\pi} \left(\frac{I_{\mathrm{E}}(\theta, \varphi)^2}{\varepsilon\omega} + \frac{I_{\mathrm{M}}(\theta, \varphi)^2}{\mu\omega} \right) (\cot\theta)^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi. \tag{32}$$

But, as is well known, the factor $\sin\theta \, \mathrm{d}\theta \, \mathrm{d}\varphi$ is the differential surface element along the φ and θ directions of a unit spherical surface. Hence, the rest of the integrand of (32) represents the radiated power per unit surface along the directions φ and θ. Thus the radiation pattern function can be evaluated directly from Fourier space components, without any inversion to the real space.

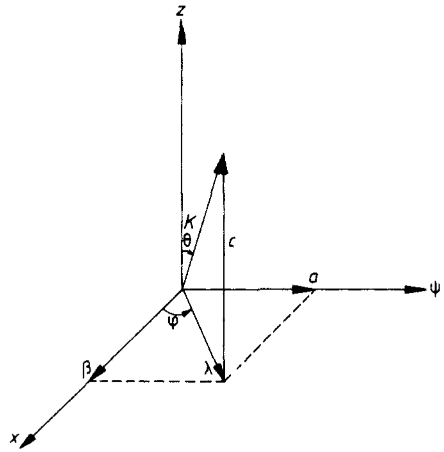

Figure 4. The direction vectors of a harmonic wave in orthogonal coordinates and in spherical coordinates.

For the verification of the method we will apply the above theory to well known cases.

(*a*) *Hertz dipole in the air*

The configuration and the equivalent electric circuit of the problem are shown in figure 5. The equivalent electric source is given by

$$V_{\mathrm{E}}(\alpha, \beta) = \mathrm{i}I_z(\alpha^2+\beta^2)\Delta l/\varepsilon\omega \tag{33}$$

and

$$z_{\mathrm{AIR}} = V/\mathrm{i}\varepsilon\omega$$

so that the equivalent electric current $I_{\mathrm{E}}(\alpha, \beta)$ can be evaluated as

$$I_{\mathrm{E}} = -I_z\Delta l(\alpha^2+\beta^2)/2\gamma. \tag{34}$$

Using now expression (32) for the radiation pattern, we have

$$P_{\mathrm{R}}(\theta) = \frac{K}{8\pi^2}\frac{|I_z\Delta l|^2}{4|\gamma|^2\varepsilon\omega}\lambda^4(\cot\theta)^2 \qquad \lambda < K \tag{35}$$

and finally

$$P_{\mathrm{R}}(\theta) = \frac{JK^2(I_z\Delta l)^2\sin^2\theta}{3\pi^2} \tag{36}$$

where J is the free-space impedance.

This result is in agreement with the existing theory of Hertz dipole radiation (Jordan and Balmain 1968).

(*b*) *Dipole radiation inside the air*

We divide the electric dipole (figure 6) into an array of Hertz dipoles, which are oriented along the z axis. Each small dipole has a current $I(z)$ which will be given by

$$I(z) = \begin{cases} I_m \sin[K(h-z)] & 0 \leqslant z \leqslant h \\ I_m \sin[K(h+z)] & -h \leqslant z \leqslant 0. \end{cases} \tag{37}$$

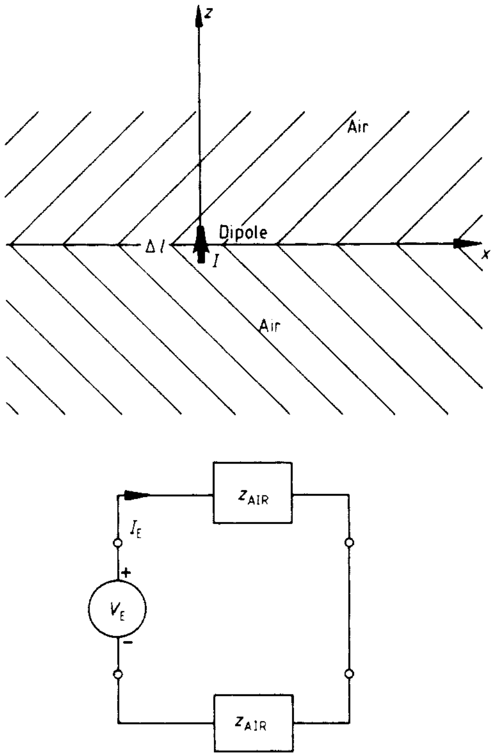

Figure 5. Electric dipole inside air and the equivalent electric circuit in Fourier space.

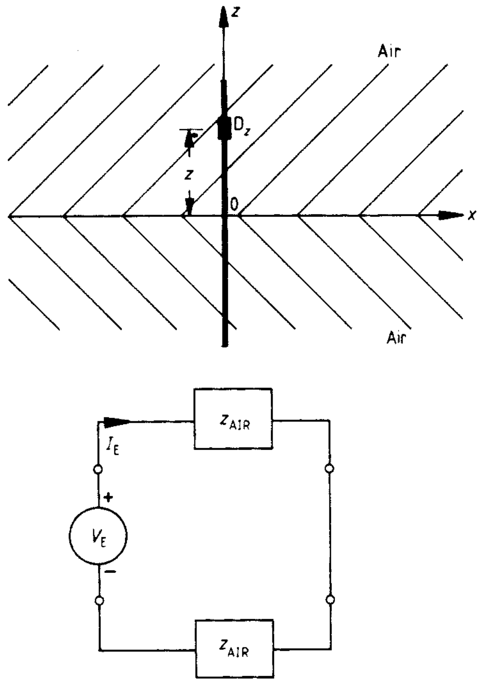

Figure 6. A long thin dipole inside air and the equivalent electric circuit in Fourier space.

Then the electric voltage $V_{\mathrm{E}}(\alpha, \beta)$ of the equivalent circuit (figure 6) can be written as

$$V_{\mathrm{E}}(\alpha, \beta)=\int_{-h}^{h} \frac{\mathrm{i} I(z)\, \mathrm{d} z}{\varepsilon \omega}(\alpha^{2}+\beta^{2})\, \mathrm{e}^{-\gamma z} \tag{38}$$

where

$$z_{\mathrm{AIR}}=\frac{\gamma}{\mathrm{i} \varepsilon \omega} \qquad \gamma=(\lambda^{2}-K^{2})^{1/2}=\mathrm{i} K \cos \theta \qquad \text{for } \lambda \leqslant K. \tag{39}$$

After some straightforward algebra, we finally have

$$V_{\mathrm{E}}(\alpha, \beta)=\frac{2 \mathrm{i} K[\cos (K h \cos \theta)-\cos (K h)]}{\varepsilon \omega} I_{m}. \tag{40}$$

Hence, the current $I_{\mathrm{E}}(\alpha, \beta)$ in the equivalent electric circuit (figure 6) will be given by

$$I_{\mathrm{E}}(\alpha, \beta)=\frac{V_{\mathrm{E}}(\alpha, \beta)}{2 z_{\mathrm{AIR}}}=\frac{-K[\cos (K h \cos \theta)-\cos (K h)] I_{m}}{\gamma}. \tag{41}$$

As a direct result, the radiation pattern function will be given by

$$P_{\mathrm{R}}(\theta)=\frac{K}{8 \pi^{2}}\left(\frac{I_{\mathrm{E}}^{2}}{\varepsilon \omega}\right)(\cot \theta)^{2}=\frac{J I_{m}^{2}}{8 \pi^{2}} \frac{[\cos (K h \cos \theta)-\cos (K h)]^{2}}{\sin ^{2} \theta}. \tag{42}$$

Expression (42) is in agreement with existing results (Jordan and Balmain 1968).

5. Evaluation of the surface modes for a dielectric slab waveguide

As a last application of this technique, we will evaluate the surface propagating modes of a dielectric slab waveguide. This problem has been solved in different ways (Unger 1977).

The proposed method provides another physical view. The configuration and its equivalent circuits are shown in figure 7. The dielectric slab separates two media with different electric parameters and the same magnetic permeability μ_0. We can prove that

$$z_{1,E}=\frac{\gamma_1}{i\varepsilon_1\omega} \qquad z_{2,E}=\frac{\gamma_2}{i\varepsilon_2\omega}$$
$$z_{S,E}=\frac{\gamma}{i\varepsilon\omega}\tanh\left(\frac{\gamma d}{2}\right) \qquad z_{P,E}=\frac{\gamma}{i\varepsilon\omega}\frac{1}{\sinh(\gamma d)}; \tag{43}$$

also

$$z_{1,M}=z_{1,E}\varepsilon_1/\mu_0 \qquad z_{2,M}=z_{2,E}\varepsilon_2/\mu_0 \qquad \text{etc}$$

with

$$\lambda^2=\alpha^2+\beta^2 \qquad K_1^2=\varepsilon_1\mu_0\omega^2 \qquad K_2^2=\varepsilon_2\mu_0\omega^2 \qquad K^2=\varepsilon\mu_0\omega^2$$
$$\gamma_1=(\lambda^2-K_1^2)^{1/2} \qquad \gamma_2=(\lambda^2-K_2^2)^{1/2} \qquad \gamma=(\lambda^2-K^2)^{1/2} \qquad K>K_1\geqslant K_2. \tag{44}$$

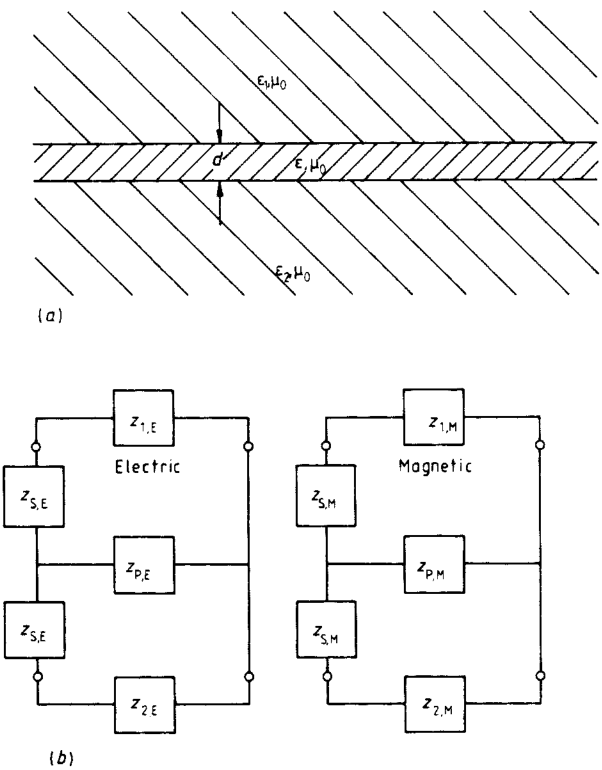

Figure 7. (*a*) A thin layer of thickness *d*, separating two different electromagnetic media. (*b*) The equivalent electric and magnetic circuits.

Let us examine what will happen as the wavenumber λ takes values from 0 to ∞.

(a) $\lambda \leqslant K_2 \leqslant K_1 < K$.

In this case, the impedances z_{1E}, z_{2E}, z_{1M}, z_{2M} of the equivalent circuits (figure 7) become real, while z_{SE}, z_{PE}, z_{PM}, z_{SM} are imaginary. The physical conclusion is that the electromagnetic energy is radiated into the surrounding media 1 and 2.

(b) $K_2 \leqslant \lambda \leqslant K_1 < K$.

z_{1E}, z_{1M} are now real, while the rest impedances are pure imaginary. Thus, the electromagnetic energy is radiated to the medium 1.

(c) $K_2 \leqslant K_1 < \lambda \leqslant K$.

All impedances of the equivalent circuits are pure imaginary. As is obvious z_{1E}, z_{2E}, z_{1M}, z_{2M}, z_{PE}, z_{PM} are of capacitive type, while z_{SE} and z_{SM} are of inductive type. In this case, radiation in the surrounding medium is excluded.

(d) $K_2 \leqslant K_1 < K < \lambda$.

All impedances are again pure imaginary, but all of them are of capacitive type. Radiation in the surrounding medium is also excluded.

Case (c) is important, because resonances can occur in the electric or magnetic equivalent circuits. The physical conclusion is that whenever a resonance occurs, then a surface wave (TM or TE) is propagated inside the dense layer separating the two optically lighter media. Hence, the eigenvalue equation which gives the surface propagating modes of this structure can be obtained from the resonance conditions of the equivalent circuits.

Taking the electric circuit which gives the electric type surface modes, the resonance condition becomes

$$(z_{SE}+z_{1E})(z_{SE}+z_{2E})/(2z_{SE}+z_{1E}+z_{2E})+z_{PE}=0. \tag{45}$$

A more compact expression can be obtained as

$$\tan(cd)=z_E(z_1+z_2)/(z_E^2-z_1z_2) \tag{46}$$

where

$$c=(K^2-\lambda^2)^{1/2} \qquad z_E=\frac{c}{\varepsilon_0\omega}$$
$$z_1=\frac{(\lambda^2-K_1^2)^{1/2}}{\varepsilon_1\omega} \qquad z_2=\frac{(\lambda^2-K_1^2)^{1/2}}{\varepsilon_2\omega}. \tag{47}$$

This gives the condition for TM modes. An analogous formula can be obtained for the TE modes.

The results derived by this method are in agreement, as expected, with already existing ones (Unger 1977), but this new technique provides a different view for the problem of surface waveguiding structures.

6. Conclusions

The method of spatial Fourier transforms and equivalent electric and magnetic circuits is shown to be a powerful mathematical tool for the treatment of electromagnetic field problems. It can also give a complementary physical view for the respective electromagnetic problems.

As became evident, the inversion into real space is possible but not necessary, because the important features of electromagnetic field problems can be calculated in Fourier space using the Parseval identity theorem. The method can be applied in a large variety of electromagnetic field problems, and although it is related to planar geometries, an extension to other geometries, such as cylindrical, is always possible (Papageorgiou *et al* 1981).

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